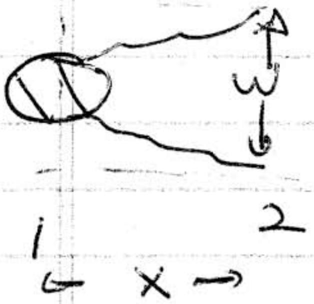
Physics 281

Lecture VI → More Wakes, Drag, [redacted]
Boundary Layers

→ Recall wake:



$F_d \approx \Delta(PA)$

$A \sim W(x)^2$

W — turbulent mixing $Re \gg 1$; viscous diffusion $Re \ll 1$

$$\frac{W}{R} \sim \left(\frac{x}{R}\right)^{1/2} Re^{-1/2}$$

$Re \ll 1$

$$\frac{W}{R} \sim \left(\frac{x}{R}\right)^{1/3} \quad Re \gg 1$$

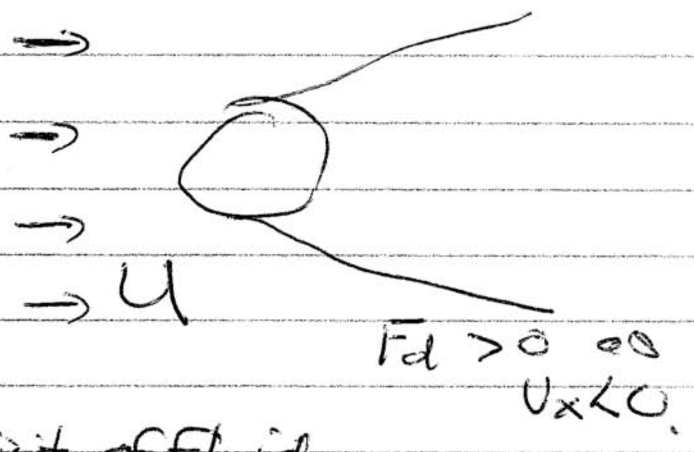
Some questions:

→ why pressure negligible? ↔ see notes

→ when does wake transition from turbulent to laminar?

Further issues/observations:

– note:



$$F_d \simeq -\int da\, \rho U v_x$$

$$\simeq -\rho U \underbrace{\int da\, v_x}$$

→ deficit of fluid flux with and without body (comparing)

flux deficit

$$Q = +\int da\, v_x$$

$$= \int da \left[\underset{\text{with}}{(U + v_x)} - \underset{\text{w/o}}{U} \right]$$

Since: $F_d = -\rho U Q$
↓
const.

Q is constant ⇒ indep. of x.

∴ potential flow outside wake must compensate for deficit

⇒ so $\int_{\text{outside}} d\underline{a} \cdot \underline{v}$ also x-indep.

$\Rightarrow v \sim 1/r^2$

$\Rightarrow \phi \sim Q/r$ $\qquad \underline{v} = \nabla \phi$

stream function,
pot flow

$\Rightarrow$ wake breaks for/aft symmetry.

Why Wake? $\Rightarrow$ Drag

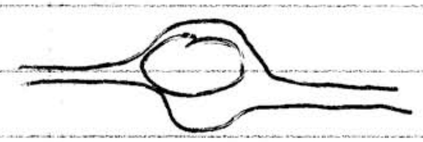
Why Drag $\Rightarrow$ separation!

Separation

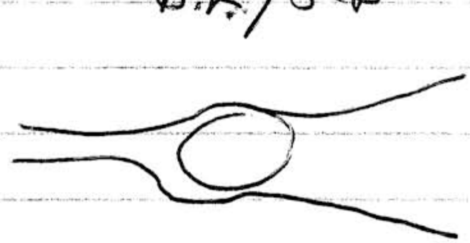
- separation occurs in boundary layer flow, which 'separates' or 'departs body'

i.e.

pot. flow

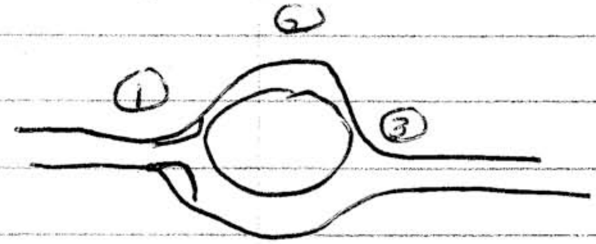


b.l./separation



separation $\Rightarrow$ wake

Why separation?



Potential Flow:

$$p + \frac{u^2}{2} = \text{const}$$

(1) $p_{tot} = p_{max} = p + \frac{u^2}{2}$ i.e. pressure maximal at stagnation point

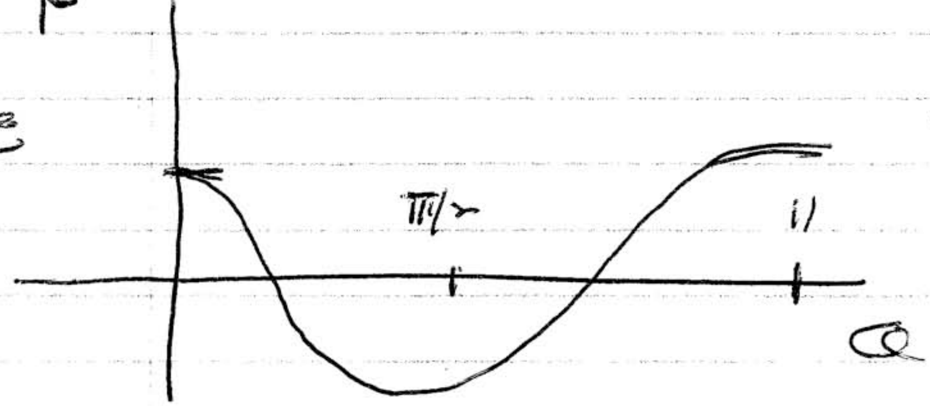
[redacted] $V_x = 0$

(2) $V_x = V_{x\,max}$ i.e. velocity relative body maximal

$p_{tot} = p_{min}$ i.e. pressure converted to flow

(3) $p_{tot} = p_{max} = p + u^2/2$ i.e. pressure returns to maximal

if sphere

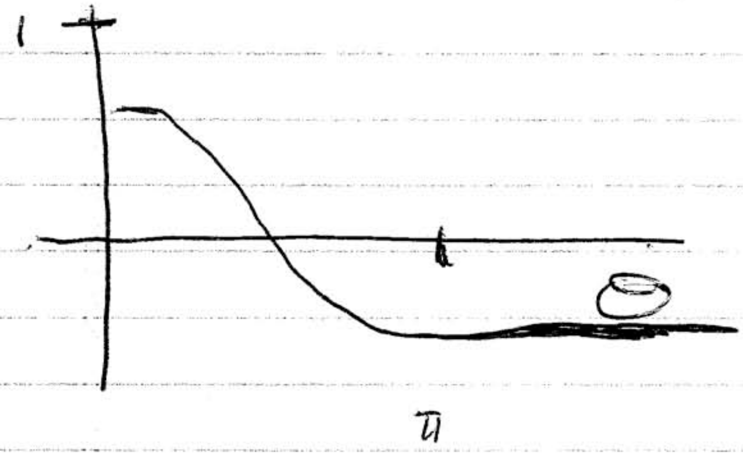


note θ symmetry

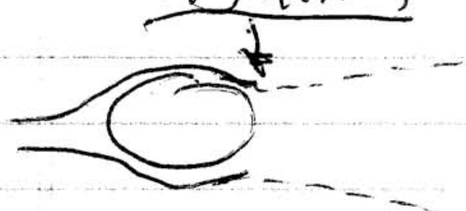
Now, if non-ideal fluid, no-slip b.c. + viscosity ⇒ fluid does not 'make it' to $\Theta = \pi$,

i.e. flow will stagnate prior

i.e.



and



stagnation ⇒ b.l. "separates" from body

⇒ opens into wake

n.b. - separated flow is unstable

⇒ driven by ∇V (Kelvin-Helmholtz).

⇒ instability driven viscous, turbulent mixing

— Simple view of instability:

① $\rightarrow$ ρ_1 high v, low p

$\rightarrow$

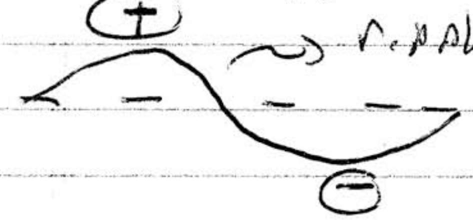
interface

ρ_2 low/zero v, high p

Ripple interface -- $\neq$ ~

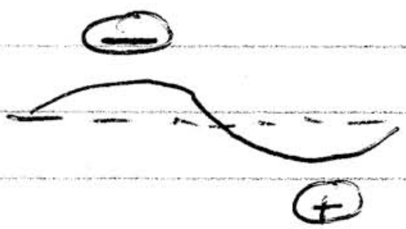
② Now, consider ripple, so tweak velocity, as shown

δv ⊕ ripple cartoon



and thus

δp



③ but $\delta p < 0$ ⇒ further $\delta v > 0$

unstable

⇒ instability drives viscous turbulent mixing, etc.

— Simple view of instability:

i.e. ⇒ $\Rightarrow$ high v, low p

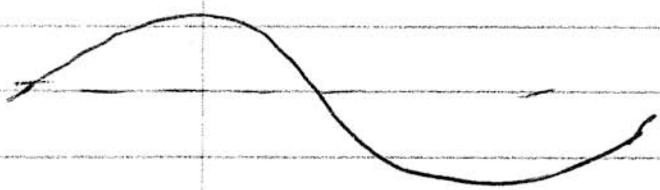
ρ_1 i.e. gradient or tangential discon.

ρ_2 low v, high p

Now: $\underline{\nabla} \cdot \underline{v} = 0$

(1) Pressure perturbation

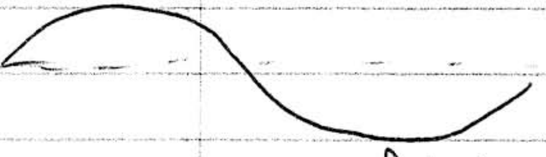
$\delta P < 0 \rightarrow$ lower



$\delta P > 0 \rightarrow$ raise

(2) Flow

$\delta V > 0$ → velocity rises in $\delta P < 0$, fast flow region



$\delta V < 0$ → velocity drops in $\delta P < 0$, slow flow region.

Point / Note:

→ δv, δp from ripple form self-reinforcing feedback loop

→ instability

→ interface treatment valid for

$$\underbrace{k}_{\text{wavelength along}} \underbrace{(\Delta z)}_{\text{layer thickness}} < 1$$

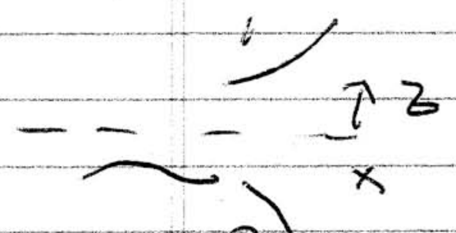
$k \Delta z \sim 1$ → need profile

Aside: To calculate:

- interface
- pot. flow
- pressure continuity

$$\left.\begin{aligned} \underline{\nabla} \cdot \underline{v} &= 0 \\ \underline{v} &= \underline{\nabla} \phi \end{aligned}\right\} \text{potential flow}$$

$$\nabla^2 \phi = 0 \Rightarrow \phi \sim \underbrace{\phi_k e^{ikx}}_{\text{wave along interface}} \underbrace{e^{-|kz|}}_{\text{damping/decay away.}}$$



2)

$$\tilde{p}_{\substack{1\\2}} = p_0 \exp\left[i(kx - \omega t) \begin{matrix} -kz \\ +kz \end{matrix} \right]$$

will need $\tilde{p}_1(0) = \tilde{p}_2(0)$.

then: $\frac{\partial}{\partial t}\underline{v} + \underline{v}\cdot\nabla\underline{v} = -\nabla p$

so

$$(-i\Omega + ikV)\,\tilde{v}_{1z} = \frac{k\tilde{p}_1}{\rho_1}$$

$$\tilde{v}_{1z} = \frac{-ik\tilde{p}_1}{\rho(kV-\Omega)}$$

Define interface by displacement:
i.e. point is displ must be same

$$\frac{d\xi}{dt} = \frac{\partial\xi}{\partial t} + v\frac{\partial\xi}{\partial x} = v_z$$

so $\quad -i(\Omega - kV)\xi = v_{1z}$

and

$$-i(\Omega - kV)\xi = \frac{-ik\tilde{p}_1}{\rho(kV-\Omega)}$$

so
$$\tilde{p}_1 = -\frac{\rho_1}{k}(kV-\Omega)^2\Sigma$$

and
$$\tilde{p}_2 = \frac{\rho_2}{k}\Omega^2\Sigma$$

equating $\Rightarrow$ $\boxed{\tilde{p}_1 = \tilde{p}_2}$

$$\Rightarrow -\frac{\rho_1}{\cancel{k}}(kV-\Omega)^2 = \frac{\rho_2}{\cancel{k}}\Omega^2$$

$$\boxed{\Omega = kV\left(\frac{\rho_1 \pm i\sqrt{\rho_1\rho_2}}{\rho_1+\rho_2}\right)}$$

$$\rightarrow \gamma \sim kV\frac{\sqrt{\rho_1\rho_2}}{(\rho_1+\rho_2)}$$

special case

$$\gamma \simeq kV/2, \quad \rho_1=\rho_2$$

i.e. $\gamma \sim k\Delta V$ — jump in velocity

→ billow formation, etc.
(see Falkovich for pics)
upshot → mix flow, relax ΔV.

→ back to wake:

- instability ⇒ vortex, wake formation

- spreading, expansion.

- viscous, turbulent mixing.

→ Separation and Boundary Layers.

- separation is bad ⇒ origin of wakes, drag

- separation is consequence of insufficient energy in B.L. flow i.e. flow can't "make it around" to rear stagnation.

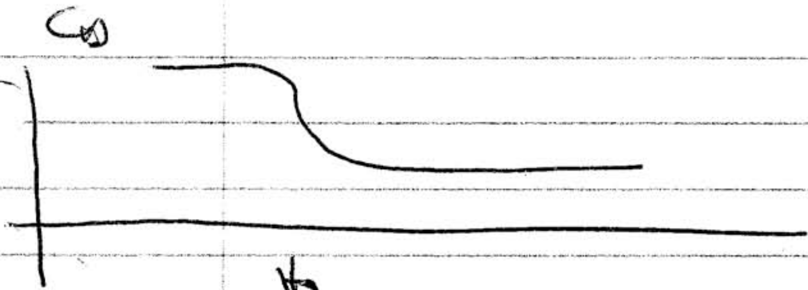
- goal is to delay separation ⇒ maintain boundary layer attachment by
  - streamlining
  - suction
  - turbulizing the B.L

How work?

- streamlining: tapering to reduce along stream pressure drop

- suction: maintain attachment by sucking BL on, vs detatchment

- turbulizing (dimples on golf ball)
  - → point is that turbulent B.L. remains attached longer (further along) by mixing energetic potential flow onto BL
  - drag with turbulent BL is lower than just before



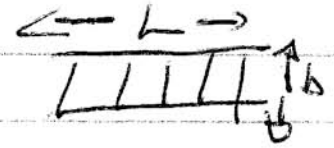
drag crisis

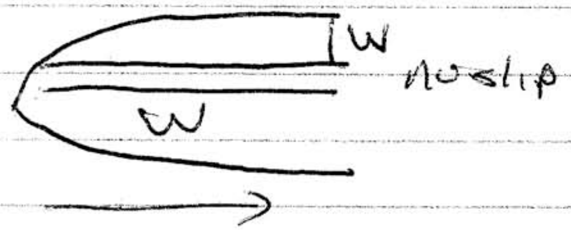
∴ this brings us to:

→ <u>Boundary Layers</u>

- need consider $\left\{\begin{array}{l}\text{viscous}\\\text{turbulent}\end{array}\right.$ BL

-- Viscous (Blasius)





$$Re \sim \frac{UL}{\nu} > 1$$

$$Re \sim \frac{vW}{\nu}$$

Plate: — Blasius BL

$$-(\nu\tau)^{1/2} \sim W \qquad \tau = \frac{x}{U}$$

$$\left(\frac{\nu x}{U}\right)^{1/2} \sim W$$

thickness

Now, for drag on plate:

$F_d \sim \int da \; -\rho\nu \frac{\partial V_x}{\partial y} \sim b$ [redacted] $\int_0^L dx\left(-\rho\nu \frac{U}{W}\right)$

skin friction

$\sim b$ [redacted] $\int_0^L dx\left(-\rho\nu \frac{U}{\left(\frac{\nu x}{U}\right)^{1/2}}\right)$

$$F_d \sim -\rho \nu^{1/2} U^{3/2} b \int_0^L \frac{dx}{\sqrt{x}}$$

$$\sim -\rho b \nu^{1/2} L^{1/2} U^{3/2}$$

$$\boxed{F_d \sim -\rho \nu^{1/2} b L^{1/2} U^{3/2}}$$

Blasius Drag Law

Compare:

- Stokes $\quad F_d \sim -\rho \nu L U$

  $Re < 1$

- Blasius $\quad F_d \sim -\rho \nu^{1/2} b L^{1/2} U^{3/2}$

  $Re_\perp < 1$

  $Re_\parallel > 1$

- Turbulent $\quad F_d \sim -\rho c_D A U^2$

  $Re > 1$

  c_D at large Re → shape, etc.

→ Can extract Blasius flow structure from approx asymptotics, a la' viscous wake.

better

→ Turbulent BL

Consider $Re \gg 1$, and $Re \gg Re_{crit}$

Now,

→ y_0 is viscous sublayer → region of transition to $V_x = 0$

→ dissipated power / volume of BL

$$P_d / V \sim \rho \frac{V^3}{y}$$

y ~ mixing length (absence scale)

(dissipn rate → indep η)

→ then, energy dissp. / time / area

$$\dot{E} \sim \int_{\underset{\hookrightarrow \text{ inner}}{y_0}}^{\delta \,\to\, \text{outer}} j \, dy \sim \int_{y_0}^{\delta} \rho \frac{V^3}{y} \, dy$$

$$\sim \rho V^3 \ln(\delta / y_0)$$

→ y_0 → laminar region → viscous sublayer

$$y_0 \sim \text{[redacted]} \, \nu / \text{[redacted]} V \sim \nu / U$$

distinction at the ln level

→ now $\underset{\substack{\downarrow \\ \text{energy/area}}}{\dot{E}} \sim \rho V^3 \ln(\delta / y_0)$

$y_0 \sim \nu / U$

<u>but</u>

$$\frac{F_d}{\text{area}} \sim \rho V^2$$

and so $\dot{E} / \text{area} \sim \overset{\substack{\text{Power} \\ \downarrow}}{\rho V^2 U} \sim F_d U / \text{area}$

⇒

$$V \sim u / \ln(\delta / y_0)$$

now $y_0 \sim \nu / u$

$\delta \sim R$ (macro-scale)

$$V \sim u / \ln\left(\frac{Ru}{\nu}\right) \sim u / \ln Re$$

$$\boxed{V \sim u / \ln Re}$$

→ $V < u$

→ Drag is flow momentum lost to body

∴ ⇒ turbulence transports momentum to body

$$\Pi_{y,x} \sim \rho \langle \tilde{v}_y \tilde{v}_x \rangle$$

$$\sim \rho V^2$$ → momentum flux

so, for profile on BL:

eddy viscosity

$$\rho \langle v_y v_x \rangle \sim \rho v^2 \sim \rho \nu_T \frac{\partial u}{\partial y}$$

N.B. Fickian ansatz debatable

diffusive flux (model)

mixing velocity

$$\nu_T = v\, l$$

$\hookrightarrow$ mixing length

eddy / turbulent viscosity

$v \to v$ (turbulent velocity)

$l \to y$ distance from wall (none other!)

$$\rho v^2 \sim \rho v y \frac{du}{dy}$$

$$du/dy \sim v/y$$

$$\boxed{u \sim v \ln(y/y_0)}$$

"Law of the wall" $\rightarrow$ Prandtl

$\Rightarrow$ logarithmic profile !

$\leftrightarrow$ strong mixing.

- ⇒ thickness of BL:

$$\frac{dy}{dx} \sim \frac{V_y}{V_x}$$

$$\Rightarrow \quad \frac{\delta}{R} \sim \frac{v}{u}$$

$$\boxed{\delta \sim R/\ln(Re)}$$

- Thickness: $\delta \sim R/\ln(Re) \ll R$

but compare to Blasius

$$\delta_{BL} \sim \left(\frac{\nu R}{u}\right)^{1/2} \sim \frac{R}{\sqrt{Re}}$$

but $\delta > \delta_{BL}$

$$R/\ln(Re) > R/\sqrt{Re} \quad \checkmark$$

turbulent BL much thicker than corresponding Laminar BL.

→ Key elements of turbulent BL:

- mixing process by momentum transport
- inertial layer and viscous sub-layer structure
- mixing length as $\sim y$, distance from wall

- balance:

$$\frac{\text{Power}}{\text{Area}} \sim \frac{F_d U}{\text{area}} \sim \int_{y_0}^{\delta} \frac{V^3}{y}$$

$$F_d/A \sim \rho V^2$$
$\Downarrow$
turb. stress

- profile of flow:

$$\rho V^2 \sim \rho V_T \frac{\partial U}{\partial y}$$
$$\sim \rho V y \, \partial U/\partial y$$

⇒ log profile

→ Pipe Flow $\quad Re \gg 1$

- similar

- V_* from: $\quad \Delta P \pi R^2 \sim \rho V_*^2 2\pi R \ell$

$$\frac{\Delta P}{\ell} \pi R^2 \sim \rho V_*^2 2\pi R$$

- $P_d \sim \rho \frac{V_*^3}{y}$

- $\rho V_*^2 \sim \nu_T \frac{\partial U}{\partial y}$

$\nu_T \sim V_* y$

PLATE 13.

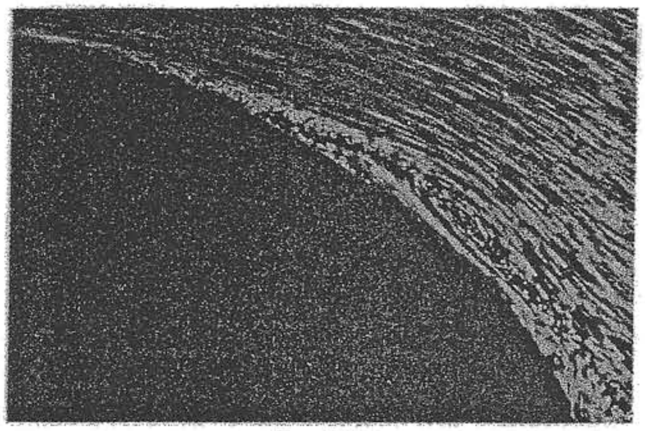


FIG. 28.

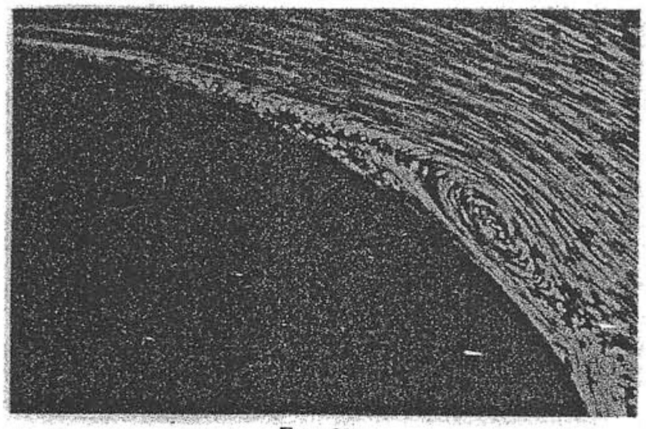


FIG. 29.

PLATE 13.—(*Continued*)

FIG. 30.

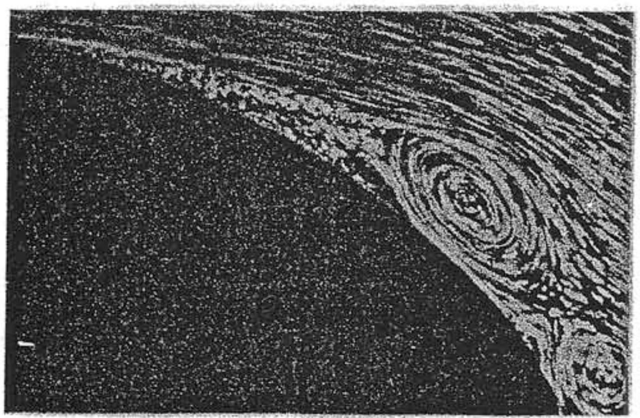


FIG. 31.

PLATE 14.

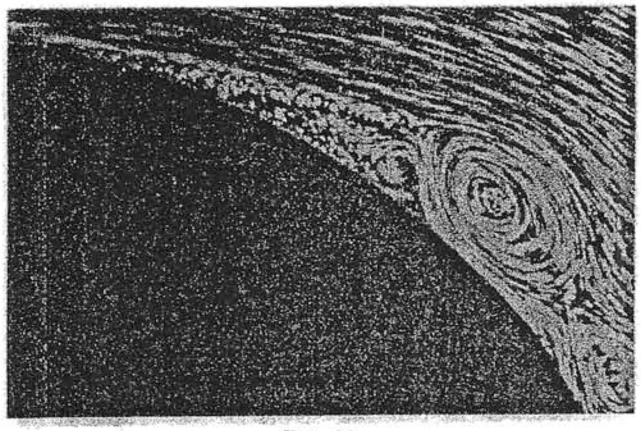


FIG. 32.

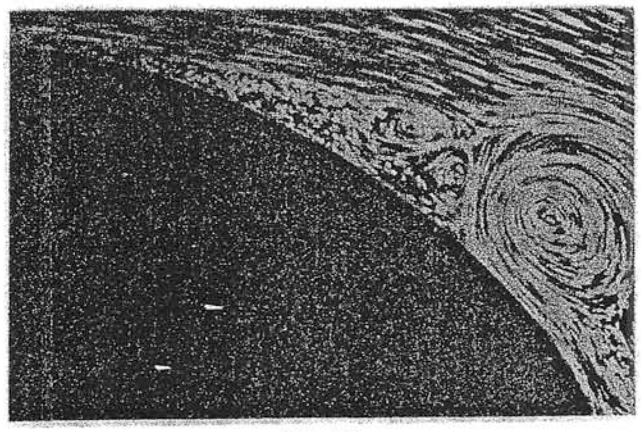


FIG. 33.

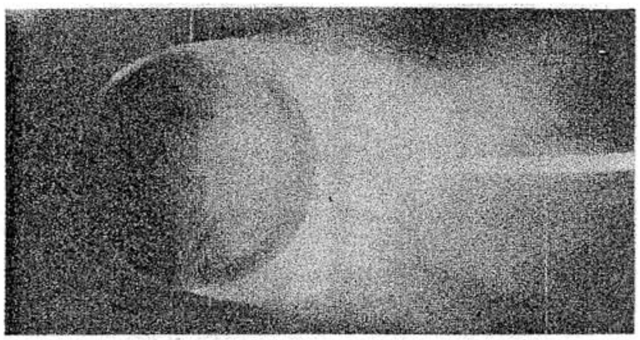


FIG. 34.—Flow round sphere below critical point. (*Wieselsberger.*)

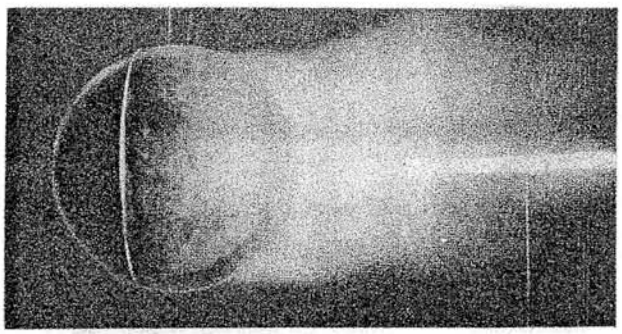


FIG. 35.—Owing to a thin wire ring round the sphere, the flow becomes of the other type with turbulent boundary layer. (*Wieselsberger.*)

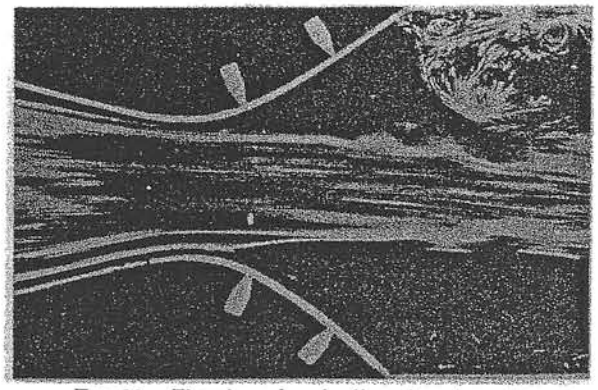


FIG. 36.—Flow in a sharply diverging channel.

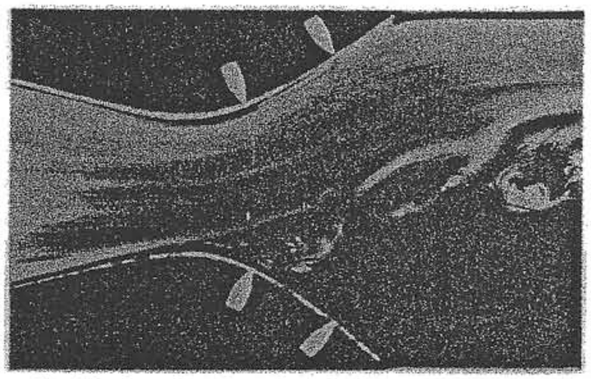


FIG. 37.—The boundary layer is sucked away at the upper wall.

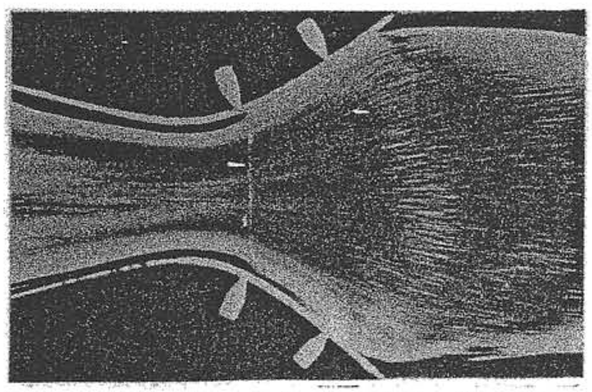


FIG. 38.—The boundary layer is sucked away on both walls; the flow is from left to right.